

Effective spacetime and Hawking radiation from moving domain wall in thin film of $^3\text{He-A}$.

T.A. Jacobson¹ and G.E. Volovik^{2,3}

¹*Department of Physics, University of Maryland, College Park, MD 20742-4111, USA*

²*Low Temperature Laboratory, Helsinki University of Technology, P.O.Box 2200, FIN-02015 HUT, Finland*

³*L.D. Landau Institute for Theoretical Physics, Kosygin Str. 2, 117940 Moscow, Russia*

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An event horizon for “relativistic” fermionic quasiparticles can be constructed in a thin film of superfluid $^3\text{He-A}$. The quasiparticles see an effective “gravitational” field which is induced by a topological soliton of the order parameter. Within the soliton the “speed of light” crosses zero and changes sign. When the soliton moves, two planar event horizons (black hole and white hole) appear, with a curvature singularity between them. Aside from the singularity, the effective spacetime is incomplete at future and past boundaries, but the quasiparticles cannot escape there because the nonrelativistic corrections become important as the blueshift grows, yielding “superluminal” trajectories. The question of Hawking radiation from the moving soliton is discussed but not resolved.

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Introduction. Condensed matter systems can serve as a useful model to study problems related to black-hole event horizons [1,2]. Recently we found that moving textures in a quantum fluid – superfluid $^3\text{He-A}$ – provide for us the possibility to study quantum properties of the event horizon, including Hawking radiation [3]. However in that example the Hawking radiation was essentially masked by Schwinger pair creation outside the horizon, which appeared to be the main mechanism of quantum dissipation at zero temperature. Here we discuss another texture, where now the Hawking radiation may dominate. This texture is a soliton, which is topologically stable in a thin film of superfluid $^3\text{He-A}$. Our work is partially motivated by recent success in experimental study of thin superfluid ^3He films, where the density of superfluid component was measured using the third sound technique [4].

Order parameter and quasiparticle spectrum. The orbital part of the order parameter of the A-phase state in thin films is characterized by a complex vector which is parallel to the plane of the film:

$$\vec{\Psi} = \mathbf{e}_1 + i\mathbf{e}_2, \quad \mathbf{e}_1 \perp \hat{\mathbf{z}}, \quad \mathbf{e}_2 \perp \hat{\mathbf{z}}, \quad (1)$$

where the axis z is along the normal to the film. This vector characterizes the Bogoliubov-Nambu Hamiltonian for the fermionic excitations in the $^3\text{He-A}$ vacuum:

$$\mathcal{H} = v_F(p - p_F) \tau_3 + \mathbf{e}_1 \cdot \mathbf{p} \tau_1 - \mathbf{e}_2 \cdot \mathbf{p} \tau_2. \quad (2)$$

where τ_a are the Pauli matrices in the Bogoliubov-Nambu particle-hole space, and we neglected the conventional spin degrees of freedom. The square of the Hamiltonian matrix $\mathcal{H}_A^2 = E^2(\mathbf{p})$ gives the square of the quasiparticle energy spectrum:

$$E^2(\mathbf{p}) = v_F^2(p - p_F)^2 + (\mathbf{e}_1 \cdot \mathbf{p})^2 + (\mathbf{e}_2 \cdot \mathbf{p})^2. \quad (3)$$

The simplest realization of the equilibrium (vacuum) state of $^3\text{He-A}$ is $\mathbf{e}_1^{(eq)} = c_{\perp} \hat{\mathbf{x}}$ and $\mathbf{e}_2^{(eq)} = c_{\perp} \hat{\mathbf{y}}$, where the parameter $c_{\perp} \sim 3$ cm/sec at zero pressure. All the other degenerate states can be obtained by the symmetry operations: continuous rotations about the axis z and discrete π -rotation about a perpendicular axis. The vacuum manifold consists of two disconnected pieces which can be transformed to each other only by the discrete transformation. This results in the topological solitons - domain walls.

Domain wall If the domain wall is parallel to the plane y, z the order parameter has the following form

$$\mathbf{e}_1(x) = c^x(x) \hat{\mathbf{x}}, \quad \mathbf{e}_2(x) = c^y(x) \hat{\mathbf{y}}, \quad (4)$$

Across the soliton either the function $c^x(x)$ or the function $c^y(x)$ changes sign: both cases correspond to the same class of topologically stable soliton and can transform to each other. The horizon appears in the former case, and for simplicity we choose the following Ansatz for such a soliton

$$c^y(x) = c_{\perp}, \quad c^x(x) = -c_{\perp} \tanh \frac{x}{d}. \quad (5)$$

It is close to the solution for the domain wall obtained in Ref. [5] (see Fig. 3b of Ref. [5]). Here the thickness of the domain wall $d \sim 1000\text{\AA}$ and is of order the thickness of the film [4].

“Relativistic” spectrum. We are interested in the low-energy quasiparticles, which are concentrated in the vicinity of momenta $\mathbf{p} = \pm p_F \hat{\mathbf{z}}$. Close to these two points the quasiparticle energy spectrum becomes:

$$E^2(\mathbf{p}) = c_{\parallel}^2(p_z \mp p_F)^2 + (c^x(x)p_x)^2 + c_{\perp}^2 p_y^2, \quad c_{\parallel} = v_F, \quad (6)$$

up to terms of order p_\perp^4/m_*^2 , where $m_* = p_F/v_F$. After shifting the momentum p_z (6) can be rewritten in the Lorentzian form

$$g^{\mu\nu} p_\mu p_\nu = 0 . \quad (7)$$

Here $p_\mu = (-E, p_i)$ is the four momentum, and the nonzero elements of the inverse metric are given by

$$g^{00} = 1 , \quad g^{zz} = -c_\parallel^2 , \quad g^{yy} = -c_\perp^2 , \quad g^{xx} = -(c^x(x))^2 . \quad (8)$$

Thus in this domain wall the speed $c^x(x)$ of light, propagating across the wall, changes sign. This corresponds to the change of the sign of one of the vectors, \mathbf{e}_1 , of the effective vierbein in Eq.(4).

Effective space-time induced by stationary soliton. The line element corresponding to the inverse metric in Eq.(8) is $(ds^2)_{3+1} = (ds^2)_{1+1} - c_\perp^{-2} dy^2 - c_\parallel^{-2} dz^2$, with

$$(ds^2)_{1+1} = dt^2 - (c^x(x))^{-2} dx^2 , \quad (9)$$

We emphasize that the coordinates t, x, y, z are the coordinates of the background Galilean system, while the interval ds describes the effective Lorentzian spacetime viewed by the low-energy quasiparticles.

The line element (9) represents a *flat* effective spacetime for any function $c^x(x)$. The singularity at $x = 0$, where $g^{xx} = 0$, can be removed by a coordinate transformation. In terms of a new coordinate $\zeta = \int dx/c^x(x)$ the line element takes the standard form $dt^2 - d\zeta^2$. With $c^x(x)$ given by (5) ζ diverges logarithmically as x approaches zero. Thus (9) is actually *two copies* of flat spacetime glued together along the line $x = 0$ where $c^x(x)$ vanishes. This line is an infinite proper distance away along any spacelike or timelike geodesic. The two spacetimes are disconnected in the relativistic approximation, however this approximation breaks down near $x = 0$ and the two halves actually communicate. One must also keep in mind that invariance under general coordinate transformations holds only for the physics of “relativistic” low-energy quasiparticles, but not for the background superfluid and high-energy “nonrelativistic” excitations. The singularity at $x = 0$ is thus physical, but is unobservable by the low-energy quasiparticles since the Ricci scalar is zero everywhere.

Effective space-time in moving domain wall. Let the soliton move relative to the superfluid: $\mathbf{v} \equiv \mathbf{v}_w - \mathbf{v}_s = v\hat{x}$, where \mathbf{v}_w and \mathbf{v}_s are the velocities of the domain wall and superfluid condensate respectively. The energy spectrum of the quasiparticles is well defined in the soliton frame where the order parameter is again stationary; it is Doppler shifted:

$$E(\mathbf{p}) = \pm \sqrt{c_\parallel^2 p_z^2 + c_\perp^2 p_y^2 + (c^x(x)p_x)^2} - p_x v . \quad (10)$$

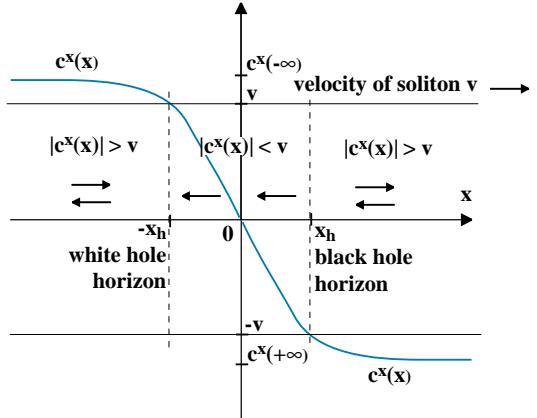


FIG. 1. The “speed of light” in the x -direction in the soliton frame, $c^x(x)$, for the singular soliton in thin films of $^3\text{He-A}$. The speed of light crosses zero at $x = 0$. For the moving soliton the black and white pair appears for any velocity v below $|c^x(\infty)|$.

This leads to the following modification of the 1+1 (t, x) metric elements:

$$g^{xx} = v^2 - (c^x(x))^2 , \quad g^{00} = 1 , \quad g^{0x} = -v . \quad (11)$$

Here now x is the “comoving” coordinate, at rest with respect to the soliton. The line element in this 1+1 effective metric takes the form:

$$(ds^2)_{1+1} = dt^2 - (c^x(x))^{-2} (dx + vdt)^2 , \quad (12)$$

which also follows directly from the Galilean transformation to the moving frame.

With $c_\perp > v > 0$ the effective spacetime geometry is no longer flat but rather that of a black hole/white hole pair (Fig.1). The black hole and white hole horizons are located where $|c^x(x)| = v$, at positive and negative x respectively:

$$\tanh \frac{x_h}{d} = \pm \frac{v}{c_\perp} . \quad (13)$$

The positions of the horizons coincide with the positions of the ergoplanes, since the metric element

$$g_{00} = 1 - \frac{v^2}{(c^x(x))^2} . \quad (14)$$

crosses zero at the same points as g^{xx} .

It follows from Eq.(13) that if v approaches the asymptotic value c_\perp of the speed of light, i.e. $c_\perp - v \ll c_\perp$, the positions of the horizons move far away from the $x = 0$ line: $|x_h| \gg d$. At these positions the gradients of the order parameter are small, so that the quasiclassical spectrum in Eq.(10) and thus the description in terms of the effective metric can well be applied near the horizons,

even if the thickness of the soliton d is of order ξ , the coherence length.

The line $x = 0$ is now at *finite* proper time or distance along some geodesics (although still at infinite proper distance along a $t = \text{constant}$ line). For example, $x = -vt$ (a point at rest with respect to the superfluid) is a geodesic along which t is the proper time, which is clearly finite as $x = 0$ is approached. Moreover, $x = 0$ is now *spacelike*, and the curvature diverges there. It is therefore akin to the singularity at $r = 0$ inside a spherically symmetric neutral black hole.

The Ricci scalar for the line element (12) is (we removed index x in c^x)

$$R = 2 \frac{v^2}{c^2} (cc'' - 2(c')^2) = -\frac{4v^2}{d^2} \left(\frac{c_\perp^2}{c^2} - \frac{c^2}{c_\perp^2} \right). \quad (15)$$

As $x \rightarrow 0$ this diverges like $-(2v/x)^2$ for any nonzero v . For $v = 0$, the spacetime is flat, as noted above. At the horizon $R = -(2c_\perp/d)^2(1 - (v/c_\perp)^4)$. Note that as $v \rightarrow c_\perp$ the curvature at the horizon goes to zero.

The positive x piece of (12) has the causal structure of regions I and II of the Penrose diagram (Fig. 2(a)) for the radius-time section of a Schwarzschild black hole, while the negative x piece has the structure of regions III and IV. These two pieces fit together as shown in Fig. 2(b).

The causal diagram reveals that the physical ranges of the coordinates t and x do not cover a complete manifold in the sense of the line element (12). Geodesics of finite length can run off the thin dashed line boundary at the lower edge of region I or the upper edge of region III in 2(b). This at first appears paradoxical: how could a quasiparticle escape from physical space and time in a finite “proper” time (or affine parameter along a lightlike geodesic)? The answer is that the energy in the superfluid frame would diverge at the edge where t goes to infinity (due to the “gravitational blueshift”), but before it actually diverges the higher order terms in the dispersion relation (3) become important, and the quasiparticle is deflected from the trajectory governed by the metric (12). For example, if we follow an outgoing q.p. backwards in time towards the horizon, relativistically it would run off region I into region IV in Fig. 2(a). In fact, however, as it gets close to the horizon, its momentum grows until (10) no longer holds. The higher order term $p_\perp^4(v_F/2p_F)^2$ in the dispersion relation (3) gives the q.p. a “superluminal” group velocity $v_F p_\perp/p_F > c_\perp$ at large p_\perp , so it crosses the horizon backwards in time and runs into the singularity. Whether it survives this encounter with the singularity we do not yet know.

Quantum dissipation by Hawking radiation. In the presence of a horizon the vacuum becomes ill-defined, which leads to dissipation during the motion of the soliton. One of the mechanisms of dissipation and friction may be the Hawking black-body radiation from the hori-

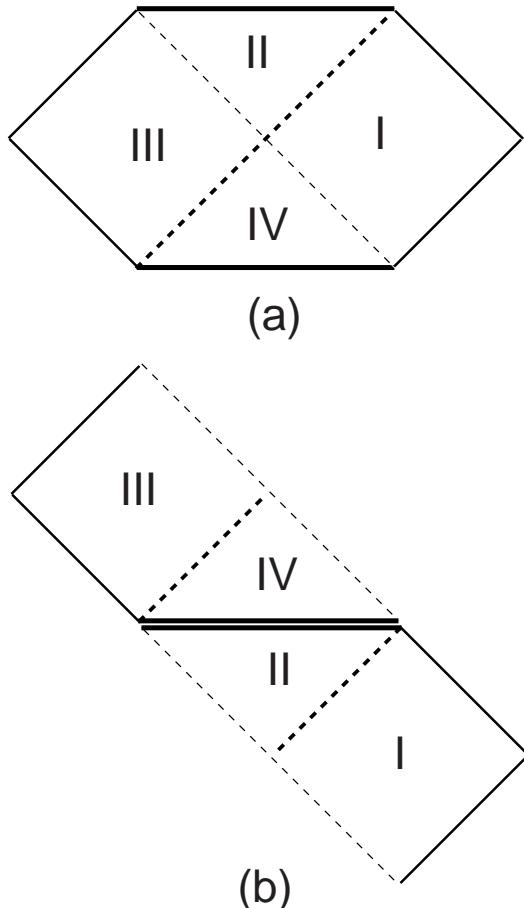


FIG. 2. Penrose diagrams. The horizontal thick solid lines represent curvature singularities. The solid lines at 45° are at lightlike infinity. Fig. (2a) is the diagram for a spherically symmetric neutral (Schwarzschild) black hole. The thick dashed line is the black hole horizon and the thin dashed line is the white hole horizon. Fig. (2b) is the Penrose diagram for the moving thin film soliton, which can be constructed by cutting Fig. (2a) along the thin dashed line and glueing the two pieces together along the singularity. Here the thick dashed lines are the black and white hole horizons at $x = \pm x_h$, while the thin dashed lines are boundaries where the effective relativistic spacetime is incomplete. However, the diagram represents all of physical space and time (see text).

zon [6], with temperature determined by the “surface gravity”:

$$T_H = \frac{\hbar}{2\pi k_B} \kappa, \quad \kappa = \left(\frac{dc^x}{dx} \right)_h. \quad (16)$$

In our case of Eqs.(5,12) the Hawking temperature depends on the velocity v :

$$T_H(v) = T_H(0) \left(1 - \frac{v^2}{c_\perp^2} \right), \quad T_H(0) = \frac{\hbar c_\perp}{2\pi k_B d}. \quad (17)$$

Typically $T_H(0) \sim 1 \mu K$, however we must choose v close to c_\perp to make the relativistic approximation more reli-

able. This Hawking radiation could in principle be detected by quasiparticle detectors. Also it leads to energy dissipation and thus to deceleration of the moving domain wall even if the real temperature $T = 0$. Due to deceleration caused by Hawking radiation, the Hawking temperature increases with time. The distance between horizons, $2x_h$, decreases until the complete stop of the domain wall when the two horizons merge (actually, when the distance between them becomes comparable to the “Planck length” ξ). The Hawking temperature approaches its asymptotic value $T_H(v = 0)$ in Eq.(17); but when the horizons merge, the Hawking radiation disappears: there is no more ergoregion with negative energy states, so that the stationary domain wall is nondissipative, as it should be.

At the moment, however, it is not very clear whether and how the Hawking radiation occurs in this system. There are several open issues: (i) The particle conservation law may prevent the occupation of the negative energy states behind the horizon. (ii) Even if these states can be occupied they may saturate, since the quasiparticles are fermions, thus cutting off further Hawking radiation. (iii) The appropriate boundary condition for the Hawking effect may not hold: the outgoing high frequency modes near the horizon should be in their ground state. These modes come from the singularity, since they propagate “superluminally”, so the physical question is whether, as the singularity moves through the superfluid, it excites these modes or not. (iv) Even if this boundary condition holds initially, the mechanism discussed in [7] of runaway damping of Hawking radiation for a superluminal fermionic field between a pair of horizons may occur. (v) Although the Hawking temperature (17) can be arbitrarily low for v near c_\perp , this is the temperature in the frame of the texture. Perhaps more relevant to the validity of the relativistic approximation is the temperature in the frame of the superfluid, which is given by $T_{sf} = T_H(0)(1 + v/c_\perp)$. This is never lower than $T_H(0) \sim \hbar c_\perp/d \sim m_* c_\perp^2$ if $d \sim \xi$, which is just high enough for the nonrelativistic corrections to be important near the peak of the thermal spectrum. Thus, unless the width of the soliton d can be arranged to be much greater than the coherence length ξ , only the low energy tail of the radiation will be immune to nonrelativistic corrections. We leave these problems for further investigation.

We expect that as $v \rightarrow 0$, the entropy of the domain wall approaches a finite value, which corresponds to one degree of quasiparticle freedom per Planck area. This is similar to the Bekenstein entropy [8], but it comes from the “nonrelativistic” physics at the “trans-Planckian” scale. It results from the fermion zero modes: bound states at the domain wall with exactly zero energy. Such bound states, dictated by topology of the texture, are now intensively studied in high-temperature superconductors and other unconventional superconductors.

tors/superfluids (see references in [9] and [10]). When $v \neq 0$, there must be another contribution to the entropy, which can be obtained by integrating $dS = dE/T$.

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